

Chapter

# 3

## Forces on a 2D Plane

---

**In this chapter, you will learn the following to World Class standards:**

- 1. Measuring the Magnitude and Direction of a Force Vector**
- 2. Computing the Resultant Force Vector from Two Forces**
- 3. Computing the Resultant Force Vector from Three Forces**
- 4. Computing the Resultant Force Vector with a Negative Force**

## Measuring the Magnitude and Direction of a Force Vector

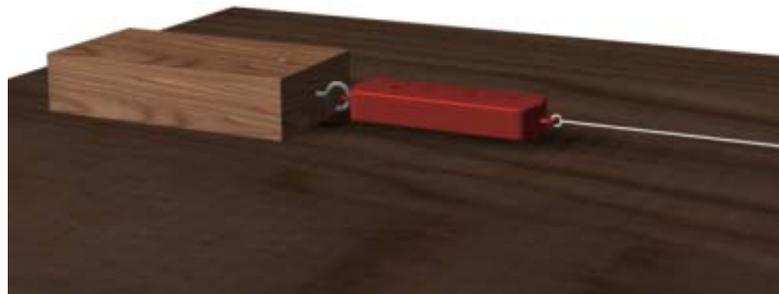
---

When studying how forces act on an object, a Computer Aided Design (CAD) application program can be very useful in determining the overall magnitude and direction of the resulting force on the entity. In previous years, individuals having knowledge of this Engineering technique would find each subcomponent leg of a force that fell on the X, Y and Z axis and they could be added or subtracted based upon their direction and therefore deriving a total answer in all three axis, the resultant force acting on the body is then found. The long process to locate the sub-vectors involves the using Trigonometric functions such as Sine, Cosine and Tangent. To set up such a mathematical problem requires us to visualize the problem, so we can use the correct formula to evaluate the force distribution. Any errors in sketching, from poor hand and eye coordination ending in a sloppy illustration, or from a minor flaw in the mathematical calculation would give the designer an incorrect conclusion. In our experience, many professionals, avoid the lengthy computations and send them out to other experts. Many project managers appreciate visual design and have never really enjoyed the hand written math problems many of us are taught in school. The fast techniques we are about to show are error free and have been used for decades.

When we were taught Mechanics in the last part of 20<sup>th</sup> century but prior to the presentation of the Personal Computer in every workspace, we learned how to graphically confirm our computations as a check for the more precise written mathematical approach. After doing the math to determine the resulting forces on an entity, we would begin a process using graph paper to draw each force vector's magnitude and direction to scale, so we could attach each vector from head to tail and compute a resultant. Depending on the drafting skills of the designer, the answer shown on the diagram could be very close to the calculations on our paper. This was a useful checking tool for any mathematical calculation.

After the advent of personal computing with inexpensive 2D and 3D design software, the previously semi-accurate graphical method of computing total force outcomes is now more accurate than the figuring out the answer using a calculator along with paper and pencil. The individual using the calculator will want to round off answers whether on paper or by the number of digits on the Liquid Crystal Display. In the classroom, we find that the person doing the computing on paper has an accurate answer to the problem, but not as precise as the graphical solution given by the CAD program.

In these exercises, we encourage one to construct the diagrams in the CAD drawing file, and whenever do the experiment. Obtain a set of force meters, wooden blocks and some cables made of rope or heavy twine. In our lab, we can copy the setup shown in the CAD diagram and by experimentation confirm the drawing results.



**Figure 3.1 – Conducting the Experiment**

In the many years of design, many of us can remember seeing some individual in the corporate test lab, breaking hundreds or maybe even thousands of parts to determine the correct dimension for a part. We encourage testing in the lab to confirm your measurements, but if those individuals would have retained the ability to compute forces and to calculate tension, compression and shear, they would have saved their organization thousands of dollars and many hours of product development had they first been able to test the part using a computer model. Now on to the first exercise.

Let us take a block of wood that is 7 inches by 3.5 inches and nail the piece to the top of a table. Now, fasten a hook to the block of wood and to the fastener attach a force meter and pull cable and tug the block with a 20 pound force. The gauge will show the force that we are pulling on the block as shown in Figure 3.2. We can see that there is only one force acting on the entity and we can determine the magnitude and direction of the force.

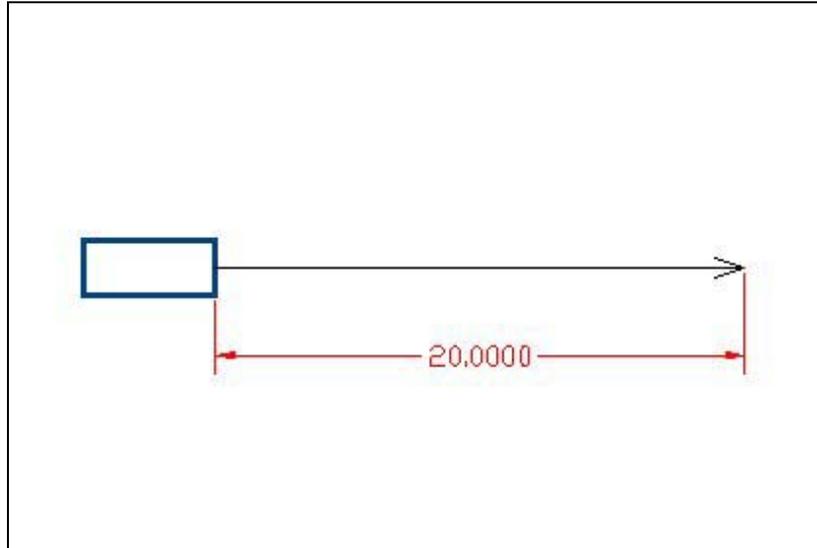


Figure 3.2 – Single Vector Acting on an Object

We can draw the block in our CAD program and attach a line with the exact length of 20. Place the arrow on the end, so we can visualize that this is a force vector. Now since the angle is zero in relationship to the X-axis, then we do not need to show the angular measurement in our drawing.

## Computing the Resultant Force Vector from Two Forces

**So, what happens to the wood block if a 20 pound force and a 10 pound force is applied in the same direction?** Take another 7 inch by 3.5 inch wood block and attach two force meters and pull cables onto the hook. On the opposite side of the second block, attach the force meter and cable from the first block.

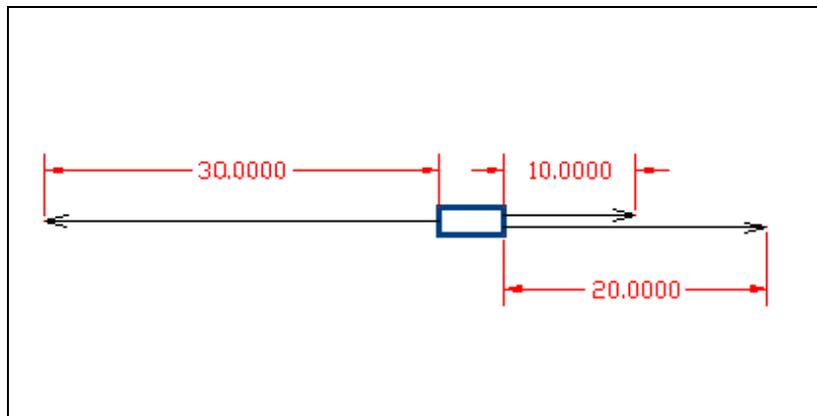
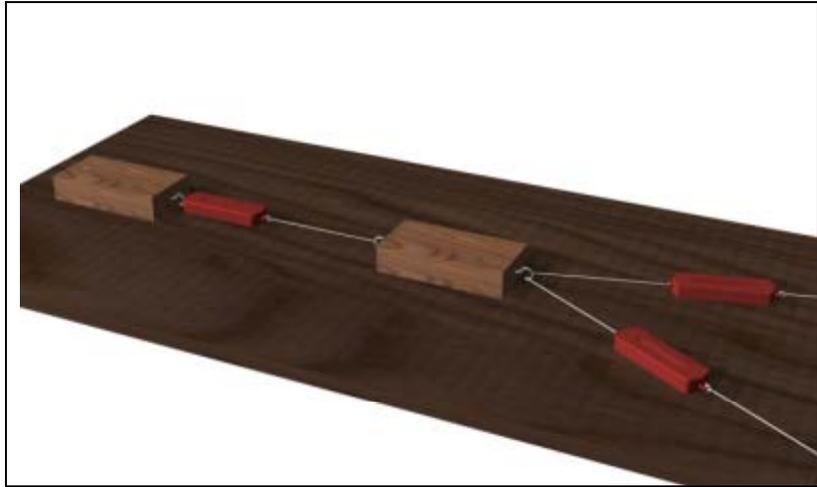


Figure 3.3 – Two Force Vectors Acting in One Direction

Now, pull with a 20 pound force on the first cable and have another individual draw on the other line with a 10 pound force. We can look at the reaction of the two forces by reading a 30 pound mark on the force meter attached on the opposite side. We are using a principle which demonstrates that for any action (force) being applied, another action (force) equal and opposite of the first will occur. Using Isaac Newton's Third Law of Motion, in Figure 3.3, we see that the two forces, the 20 and 10 pound pull were added together to 30 pounds on the one side and caused our gauge to record 30 pounds on the other. We could have substituted a resultant force vector of 30 pounds for the two separate 10 and 20 pound forces.

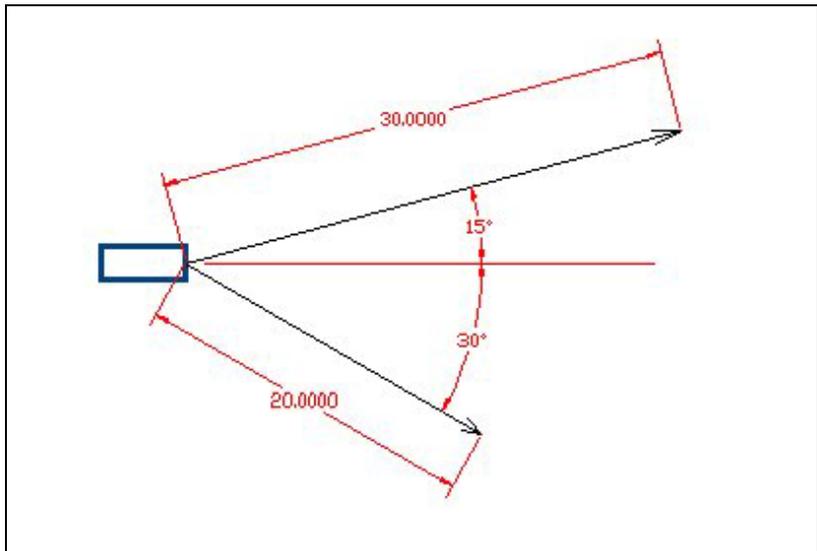
In Figure 3.4, we can observe that by pulling on the cables attached to the block on the right with the exact amount of force, the cable on the block to the left will react to the forces and the gauge will show the precise magnitude of the two forces acting on a single body and the direction of the resulting force is 180 degrees opposite of the Resultant shown in the CAD diagram.



**Figure 3.4 – Experiment with Multiple Forces**

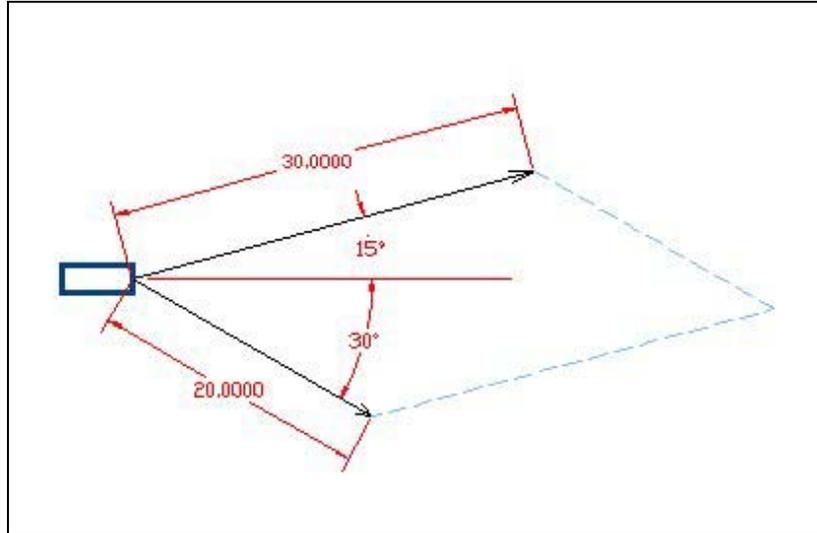
We can observe this in the experiment shown above, but can we model the problem in the CAD program and get the exact answer?

**Now, how do we handle two forces going in different directions?** First, we will present ourselves with another Mechanics problem. On our second block with two meters attached, we will lay a protractor on the table and pull a 30 pound force on one cable at 15 degrees above the X-axis and on the other line we will pull a 20 pound force at a minus 30 degree angle below the X-axis. Our CAD drawing will be drawn as shown in Figure 3.5.



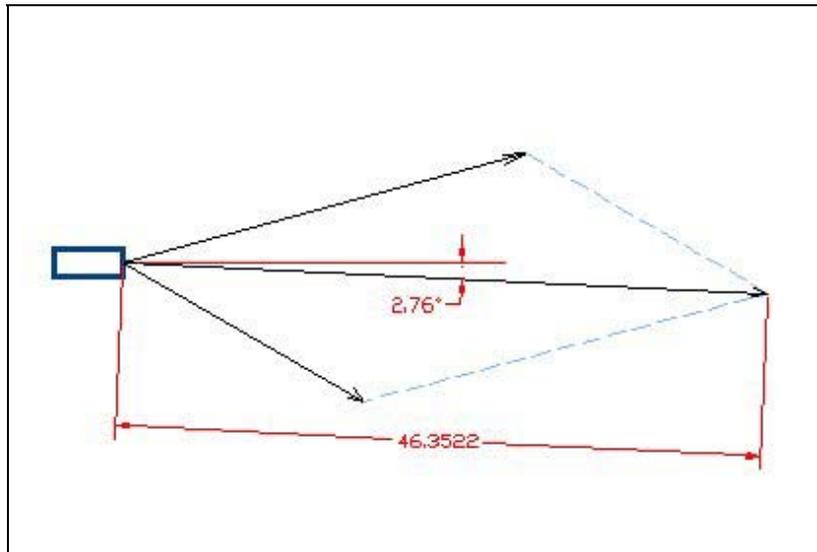
**Figure 3.5 – Two Force Vectors Acting on an Object**

**What will be effect of the two forces on the object?** In the last problem, we placed the tail of the second vector on the head of the first vector and we created a 30 pound vector at 0 degrees. In this exercise, we can add the 20 pound force vector to the 30 pound force vector. In Figure 3.6, the blue dotted lines show how the pathway of either vector added to the other will result in the same position in the drawing.



**Figure 3.6 – Using Vector Addition**

Create a new force vector on the CAD drawing starting at the base of the two forces and ending at the intersection of the two blue dotted lines. This new force vector is called the Resultant. When we measure the Resultant, we compute a 46.3522 pound force at 2.76 degrees below the X-axis as shown in Figure 3.7. Check the force meter attached to the nailed down block and read the gauge. Checks out to be exact, right.



**Figure 3.7 – Computing the Resultant Vector**

So we learn that adding force vectors is as simple as placing the tail of the subsequent vector acting on the object on the head of the preceding force vector.

**\* World Class CAD Challenge 10-1 \* - Draw a 7 inch by 3.5 inch rectangular block and illustrate a 30 pound force vector at 15 degrees above the X-axis and a 20 pound force vector at 30 degrees below the X-axis. Add one force vector onto the other to compute the Resultant. Measure the magnitude and direction of the Resultant. Save the drawing as Adding Vectors.dwg**

**Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.**

## Computing the Resultant Force Vector from Three Forces

When solving a Mechanics problem for three or more force vectors, just continue with the vector trail by concatenating the head of the first to the tail of the second and onward. In the next exercise, we will show a 32 pound force at 45 degrees above the X-axis, a 25 pound force at 10 degrees above the X-axis and 17.5 pound force at 40 degrees below the X-axis as shown in Figure 3.8.

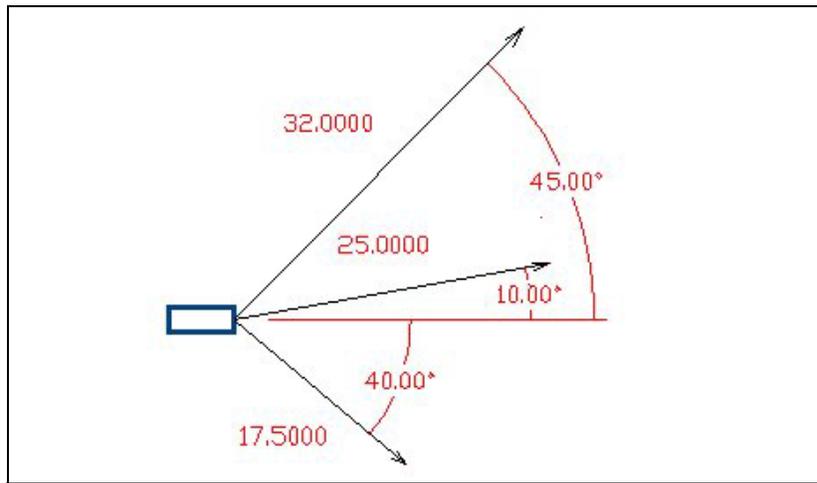


Figure 3.8 – Three Force Vectors Acting on an Object

In Figure 3.9, we copied the three force problem to just the right of the setup diagram and moved each force representation to perform vector addition. We changed the black vector lines to the dotted blue lines and removed their arrow heads. Now when we draw the Resultant, we can see the three subcomponent vectors in blue and the answer of 60.6534 pounds at 14.53 degrees above the X-axis.

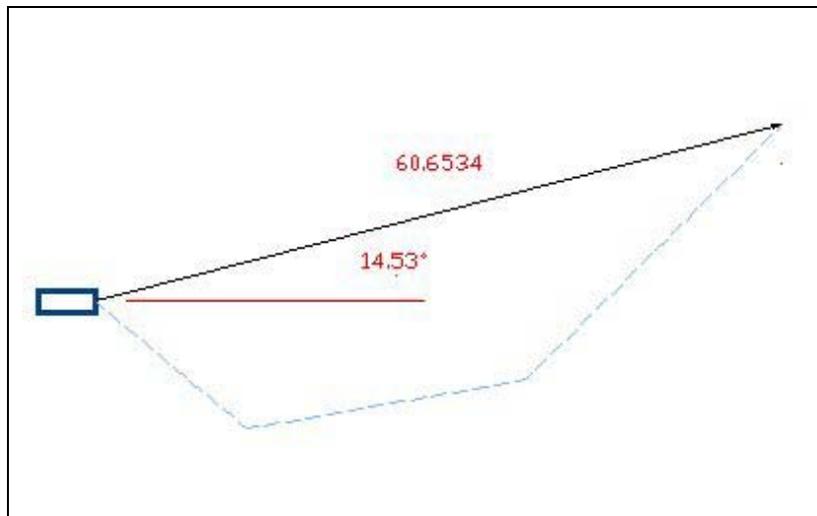
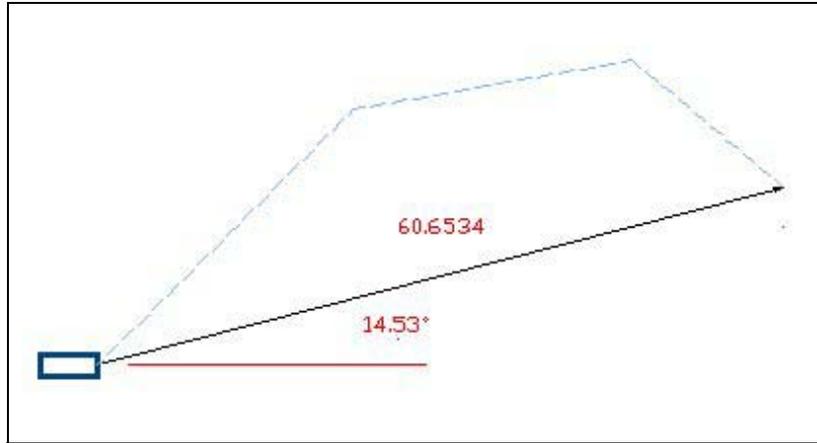


Figure 3.9 – Using Vector Addition

In complex CAD Mechanic diagrams that involve three or more force vectors, we always have two illustrations to show. The first view being the problem setup which shows the object with each force acting upon the entity. The second layout displays the vector addition pathway in dotted lines and then the resulting force vector with the computed magnitude and direction.

**A CAD Note:** We can turn off the dimension lines, arrows and the dimension extension lines, since these images can appear on the drawing similar to those that appear in a mathematical exercise versus on an ANSI or ISO controlled document, where dimensional appearances are prescribed by regulation. In Figure 3.7, we turned off the many of the dimensional attributes, except the actual dimension. Dimensioning procedures are the same.

In Figure 3.10, we constructed the vector addition problem in a different direction, and the resulting force vector or Resultant is exactly the same measurement in magnitude and direction as going in any other manner of attaching the three subcomponent force vectors. We can try every method to attach the three forces and arrive at the same answer.



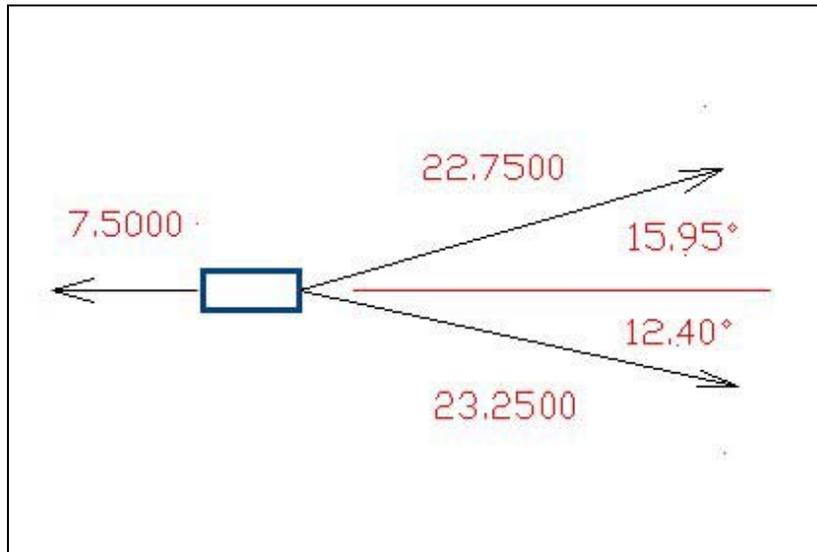
**Figure 3.10 – Vectors Add in Any Direction**

**\* World Class CAD Challenge 10-2 \* - Draw a 7 inch by 3.5 inch rectangular block and illustrate a 32 pound force vector at 45 degrees above the X-axis, a 25 pound force vector at 10 degrees above the X-axis and a 17.5 pound force vector at 40 degrees below the X-axis. Add one force vector onto the other to compute the Resultant. Measure the magnitude and direction of the Resultant. Save the drawing as Adding Vectors 2.dwg**

**Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.**

## **Computing the Resultant Force Vector with a Negative Force**

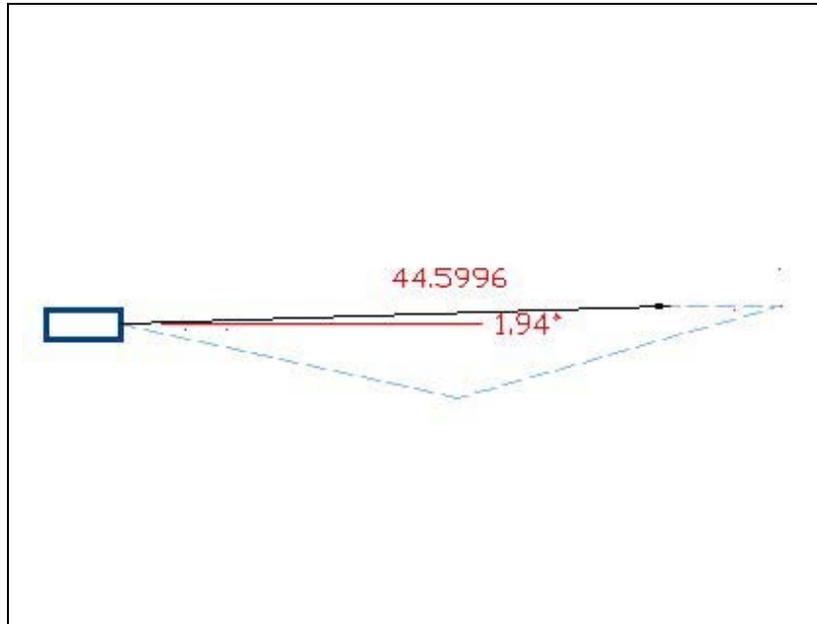
In the next exercise, we will examine the effect of a negative force or one operating in the opposite direction from the other force vectors and decide whether vector addition will work with the so-called negative force. In the exercise, we will show a 22.75 pound force at 15.95 degrees above the X-axis, a 23.25 pound force at 12.40 degrees below the X-axis and 7.5 pound force at 180 degrees around the X-axis as shown in Figure 3.11.



**Figure 3.11 – Negative Force Vector Acting on an Object**

As we may already guess, that when we add another cable and force meter to our table experiment and pull back with a force of 7.5 pounds, that the vector addition rules will still apply. We just add the so-called negative force vector onto the preceding vector.

In Figure 3.12, we copied the three force problem to just the right of the setup diagram and moved each force representation to perform vector addition. We changed the black vector lines to the dotted blue lines and removed their arrow heads. Now when we draw the Resultant, we can see the three subcomponent vectors in blue and the answer of 44.5996 pounds at 1.94 degrees above the X-axis. The 7.5 pound force reduced the magnitude and slightly altered the direction of the resulting force.



**Figure 3.12 – Computing the Resultant Vector**

**\* World Class CAD Challenge 10-3 \* - Draw a 7 inch by 3.5 inch rectangular block and illustrate a 22.75 pound force vector at 15.95 degrees above the X-axis, a 23.25 pound force vector at 12.4 degrees below the X-axis and a 7.5 pound force vector at 180 degrees around the X-axis. Add one force vector onto the other to compute the Resultant. Measure the magnitude and direction of the Resultant. Save the drawing as Adding Vectors 3.dwg**

**Continue this drill four times using some forces you have determined, each time completing the drawing under 5 minutes to maintain your World Class ranking.**

After completing this chapter, do the addition 2D vector problems in Appendix A and then continue to Vector Addition with Forces Acting in 3D in chapter 3.